

# Taking Advantage of S-Parameter

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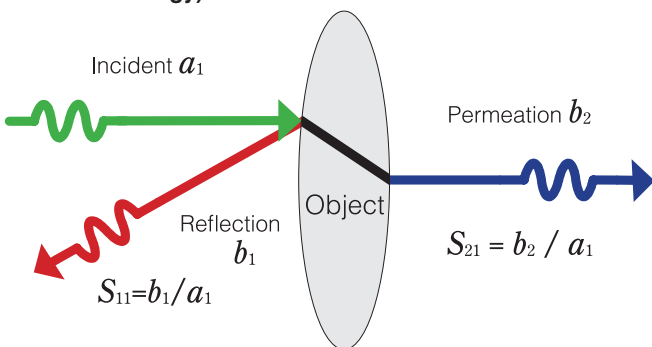
## 1 | What is the S-Parameter?

As the speeds of electronic circuits are increasing, the analog characteristics in digital circuits are becoming ever more critical. Therefore, S-parameters, which are treated differently from other parameters, are now becoming an increased center of attention again. The purpose of this chapter is to give an overview of S-parameters.

### Definition of the S-Parameter

The S-parameter (Scattering parameter) expresses circuit characteristics using the degree of scattering (i.e. reflection back to the source and transmission to other directions) when an AC signal is considered as a wave. "Degree" means how much a wave (= signal) is attenuated or amplified, while the wave is transmitted through the circuit to be evaluated. The input and output ports of a circuit are numbered, and the S-parameter that is "Incident at port  $j \rightarrow$  Detected at port  $i$ " is described as  $S_{ij}$ . If  $i = j$ , the wave is the reflection. If  $i \neq j$ , the wave is the transmission. Figure 1 shows this concept by taking the light that passes through a lens as an analogical example. For more detailed definitions, please refer to the textbooks [1-6].

Figure 1 Conceptual Diagram of S-Parameter (Optical Analogy)



The term "insertion loss", which is often used to describe the filter characteristics, can be considered almost the same as  $|S_{21}|$ . The value of insertion loss and  $|S_{21}|$  will be different if the input and output terminal conditions don't match. Even though, the insertion loss can be converted to  $|S_{21}|$  by calculation, and furthermore, this situation (= different terminal conditions) doesn't occur very often. The one thing that needs to be paid attention is that the sign must not be ignored. While the S-parameter is defined as "target/reference", the insertion loss is defined as "reference/target"; which means that S-parameter and the insertion loss are reciprocal. Therefore, in logarithmic

expression (dB unit), these values must have the opposite signs (i.e. if  $|S_{21}| = -20$  dB, the insertion loss is +20 dB). By definition, the negative sign is usually not used with the insertion loss value. On the other hand, S-parameters can be expressed with both positive and negative signs; the positive sign expresses the amplification, and the negative sign expresses the attenuation.

### Characteristics of the S-Parameter

Since the S-parameter is a ratio of electrical power (to be exact, a square root of the electrical power), it is basically a non-dimensional parameter (no unit). However, when describing the magnitude of the S-parameter, the unit "dB" is usually used with a common logarithm <sup>\*1</sup>. Table 1 shows some typical values for your reference. When the S-parameter is  $1/\sqrt{2}$  (i.e. half of the electric power is transmitted or reflected), the value in logarithmic expression is -3 dB.

Table 1 Typical S-Parameter Values

$ S_{ij} $	$20\log  S_{ij} $
1	0 dB
$1/\sqrt{2}$	-3 dB
1/10	-20 dB
1/100	-40 dB
1/1000	-60 dB

The major characteristics of the S-parameter are as follows:

- ▶ In 2-port passive circuit,  $|S_{11}|^2 + |S_{21}|^2 \leq 1$ .  
Therefore, the S-parameter of the passive device does not exceed 1 (0 dB).  
If the device is lossless, we have equality (Feldtkeller's formula). There is no loss, so the total amount of the scattering should be 100%.
- ▶ If a device is reciprocal (e.g. for passive devices, a non-unidirectional component such as an isolator or circulator),  $S_{21} = S_{12}$ .  
Generally,  $S_{ij} = S_{ji}$  (the S-matrix is symmetric). Similarly, in mixed-mode S-parameters (see the next section),  $S_{cc21} = S_{cc12}$ ,  $S_{dc11} = S_{cd11}$ ,  $S_{dc21} = S_{cd12}$ .

<sup>\*1</sup> To avoid complexity, the assumption "the absolute value of  $S_{21}$  (or,  $S_{21}$ ) is  $\circ\circ$  dB", is used here. To be exact, it should be explained as " $20\log|S_{21}| = \circ\circ$  dB". It is implied that the value with the unit dB is the value that neglected "20log" (in some cases, "10log"). For your information, the terms with "loss" (e.g. insertion loss), "attenuation", "-loss" (e.g. LCL) are originally defined as the parameters that use the unit dB. These values are not expressed in antilogarithms.

Recently network analyzers are generally used to measure S-parameters. If the data may be transferred or used for simulation using the parameters as numeric values, it is convenient to save the data in a Touchstone format text file (see Figure 2 for an example).

**Figure 2 Example of Touchstone File**

#	MHz	S	MA	R	50				
0	0.005	0	0.994	0	0.994	0	0.005	0	
0.300	0.999	-0.037	0.002	89.72	0.002	89.72	0.999	-0.023	
0.315	0.999	-0.037	0.002	89.87	0.002	89.87	0.999	-0.023	
.....									
6000	0.178	-107.7	0.972	-16.48	0.972	-16.48	0.182	-101.8	

## 2 | Using the S-Parameters

### Reference Impedance

Reference impedance is an important concept for understanding and using S-parameters. The following explains the overview of the reference impedance:

Usually we simply state that  $S_{21}$  is  $\text{OO dB}$ . However, to be exact, it should be stated that “ $S_{21}$  is  $\text{OO dB}$  when  $\Delta\Delta \Omega$  is used as the reference impedance”. Since the reference impedance is usually  $50 \Omega$ , the abbreviated expression can be used. Still, it is important to keep in mind that the S-parameter is a relative value depending on a certain reference value. In other words, the reference impedance is always necessary for acquiring S-parameters (whether by actual measurement or by simulation).

The following describes what “reference” means taking 1-port S-parameter (i.e. reflection coefficient) as an example. The relationship between the reflection coefficient and the impedance  $Z$  can be expressed as follows:

$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

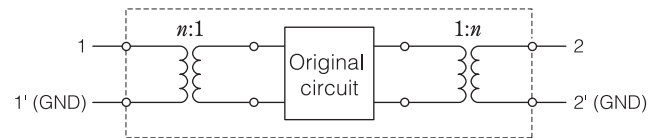
“ $Z_0$ ” in this formula indicates the reference impedance. For example, assume that there are two resistors,  $50 \Omega$  and  $200 \Omega$ . When  $50 \Omega$  is converted into the reflection coefficient using the reference impedance of  $Z_0=50 \Omega$ , we have  $\Gamma=0$ . Similarly, when  $Z=200 \Omega$ ,  $\Gamma=0.6$ . If the reference impedance of  $Z_0=200 \Omega$  is used, we have  $\Gamma=-0.6$  for  $Z_0=50 \Omega$  and  $\Gamma=0$  for  $Z_0=200 \Omega$ . In summary, the  $50 \Omega$  resistor doesn’t reflect when the reference impedance is  $50 \Omega$  (matched,  $\Gamma=0$ ), and the  $200 \Omega$  resistor doesn’t reflect when the reference impedance is  $200 \Omega$ . This means that the reflection coefficient is a relative value that varies depending on the conditions. The reference impedance is the reference value used for calculating this relative value. Non-1-port devices can be considered the same way, but the calculation is more complicated [5, 6].

Since  $50 \Omega$  is only a reference value, there are other S-parameters than  $50 \Omega$  (as described in the example above). The reference value can be changed. Suppose that you have the S-parameter already determined for  $50 \Omega$ , it is possible to transform it to a different S-parameter using a reference impedance other than  $50 \Omega$  (Note: The original S-parameters require all ( $S_{11}$ ,  $S_{21}$ ,  $S_{12}$ , and  $S_{22}$ ) parameters even when you want to know only  $S_{21}$ .). The calculation is difficult, but you can use the tools such as high-frequency simulators to obtain the calculation result easily. You can also use TDK’s “SEAT” for this calculation. Figure 4 shows S-parameters of the bead with

seriesS-thru configuration using three reference impedances of  $10 \Omega$ ,  $50 \Omega$ , and  $100 \Omega$ . The smaller the reference impedance, the higher the attenuation is. The shunt-thru capacitors, which are not shown in the figure, give the opposite, namely the bigger the reference impedance, the higher the attenuation.

**Figure 3 Varying the Reference Impedance**

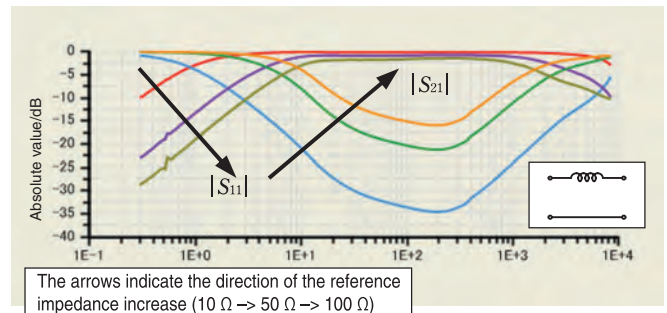
- Simulate the following circuit at the terminal  $Z_0$ .
- The ideal transformer multiplies the impedance by constant ( $n^2$ ).
- Some simulators can vary the terminal impedance freely and in such a case this workaround is not needed.



$$n = \sqrt{Z_0 / Z'_0}$$

$Z_0$  is the original reference impedance (normally  $50 \Omega$ ).  
 $Z'_0$  is the target reference impedance.

**Figure 4 S-parameter of Bead with Various Reference Impedances**



### Mixed Mode S-Parameters

This section discusses mixed-mode S-parameters. A differential transmission system is a system that uses different modes such as the differential mode and common mode (refer to the column of the last page). The S-parameter for this system needs to be handled according to the modes, which is referred to as a mixed-mode S-parameter (or modal S-parameter). The normal S-parameter (referred to as a single-ended S-parameter or nodal S-parameter) indicates the response for each port. On the other hand, a mixed-mode S-parameter indicates the response for the sum of two signals (common mode) or the response for the difference of two signals (differential mode).

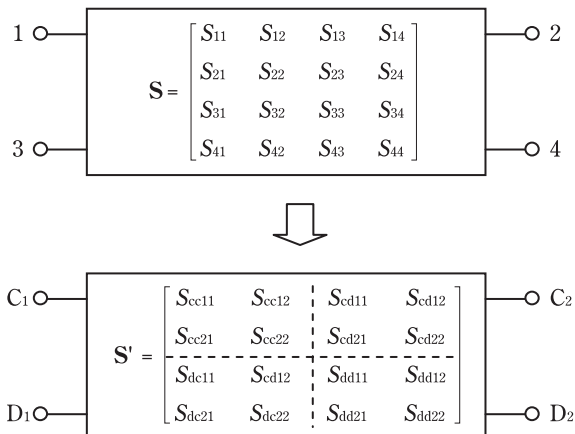
A mixed-mode S-parameter can be obtained from a single-ended S-parameter (Figure 5). The calculation formula is complicated, but you can use the simulators or other tools to obtain the mixed-mode S-parameter (Figure 6, [7]). Recently there are a lot of network analyzers that can read the mixed-mode S-parameter directly. The obtained mixed-mode S-parameters have the following meanings. See the reference [6, 8] for details.

- ▶  $S_{ccij}$ : Common mode response
- ▶  $S_{ddij}$ : Differential mode response
- ▶  $S_{dcij}$ ,  $S_{cdij}$ : Mode conversion between differential mode and common mode

The mode conversion ( $S_{dcij}$  or  $S_{cdij}$ ) can be used as the symmetric property index of the system [9, 10]. If the device has good symmetry, the mode conversion should be zero, which means that each mode is independent. LCL<sup>2</sup> can be calculated using the S-parameter as follows [7, 11, 12]:

$$\begin{aligned} \text{2-port circuit } LCL_1 &: = -20\log |2S_{dc}| = -20\log |S_{11} - S_{21} + S_{12} - S_{22}| \\ \text{4-port circuit } LCL_2 &: = -20\log |2S_{dc11}| = -20\log |S_{11} - S_{31} + S_{13} - S_{33}| \end{aligned}$$

**Figure 5 Mixed-Mode S-Parameter Can be Converted from Single-Ended S-Parameter**



**Figure 6 Calculation of Mixed Mode S-Parameter**

- Simulate the following circuit at the terminal  $Z_0$ .
- The circuits on the right and left sides convert from the single-ended to the mixed-mode (and vice versa, because the circuit is symmetric).
- Since this is an "ideal magic T" conversion circuit, it can be expressed using the ideal transformer network (hybrid coil) [7].

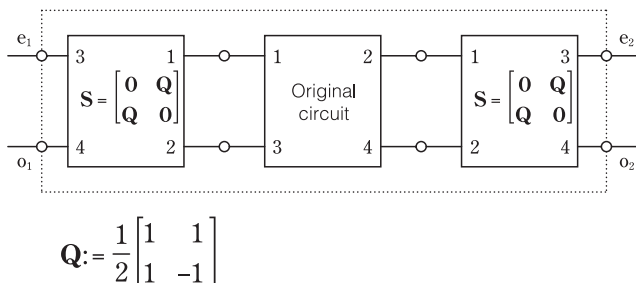
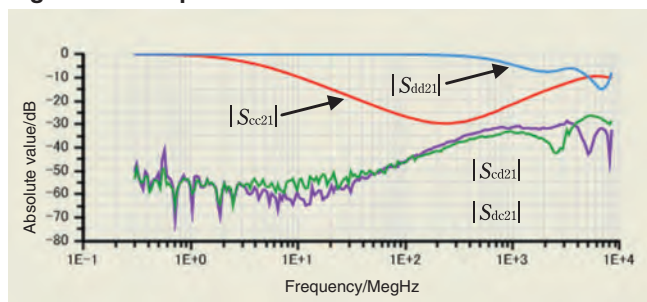


Figure 7 shows an example of actual measurements for a common mode filter (CMF). It can be seen that this CMF attenuates the common mode signal around 100 MHz ( $|S_{cc21}| = -30$  dB), but transmits the differential mode signal with little loss ( $|S_{dd21}| = 0$  dB).

**Figure 7 Example of CMF Mixed-Mode S-Parameter**



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\*2 In ITU-T Recommendation G.117-1996, "Transmission aspects of unbalance about earth", LCL is defined for 2-port and 4-port circuits. (In the Standard, 2-port and 4-port circuits are referred to as 1-port and 2-port, respectively.) To avoid confusion, LCLs for 2-port and 4-port circuits are referred to as  $LCL_1$  and  $LCL_2$ , respectively.  $LCL_2$  is equal to  $LCL_1$  at the non-target port terminated with reference impedance. Since LCL uses the signal source voltage as its reference, the difference between LCL and S-parameter is 6 dB (double). The calculation formulas in this chapter reflect this difference.

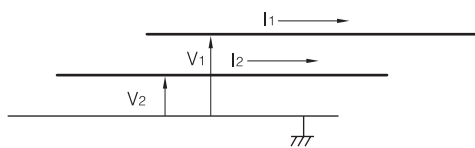
## About Common Mode

“Common mode” is one of the keywords for EMC countermeasures. This section gives details about the common mode.

### Definition of Common Mode and Differential Mode

Suppose there are two conductors (and a GND conductor) that are parallel to each other (e.g. a differential transmission system). When the voltage and the current for each conductor are named as  $V_1$ ,  $I_1$  and  $V_2$ ,  $I_2$  (see Figure 1), the common mode voltage  $V_c$  and current  $I_c$  and the differential mode voltage  $V_d$  and current  $I_d$  are defined as the following (IEC technical vocabulary, IEC60050-161:1990, JIS C0161: 1997).

Figure 1 Two Parallel Conductors



- ▶ Common mode voltage  $V_c$ : Average voltage for each conductor  
 $V_c = (V_1 + V_2) / 2$
- ▶ Common mode current  $I_c$ : Total current for each conductor  
 $I_c = I_1 + I_2$
- ▶ Differential mode voltage  $V_d$ : Voltage between two conductors  
 $V_d = V_1 - V_2$
- ▶ Differential mode current  $I_d$ : Half of the current difference for each conductor  
 $I_d = (I_1 - I_2) / 2$

The common mode represents the sum of signals, while the differential mode represents the difference between signals. The current in differential mode flows backwards (anti-phase) through two conductors. Therefore, GND is not directly connected (for that reason, it is referred to as normal mode). On the other hand, the current in the common mode flows in the same direction (in-phase) through two conductors. Consequently, the current flows through the GND conductor (or through another area) and then returns (hence this is referred to as an earth circuit).

The common mode is also referred to as asymmetrical (or vertical, as in vertical electrical current) or longitudinal, and the differential mode is also referred to as being symmetrical (or horizontal).

### Other Modes

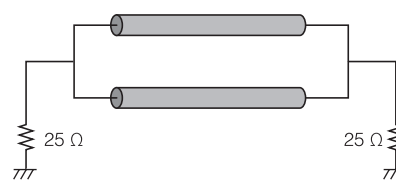
The even and odd modes are similar to the common and differential mode. Like common and differential modes, even and odd modes represent the sum of signals and the difference between signals, respectively. The difference between them is the coefficient. In the calculation formula,

- ▶ Even mode voltage  $V_e$ , Current  $I_e$ :  
 $V_e = (V_1 + V_2) / \sqrt{2}$   
 $I_e = (I_1 + I_2) / \sqrt{2}$
- ▶ Odd mode voltage  $V_o$ , Current  $I_o$ :  
 $V_o = (V_1 - V_2) / \sqrt{2}$   
 $I_o = (I_1 - I_2) / \sqrt{2}$

Compared to the common mode, the even mode voltage is  $\times\sqrt{2}$ , and the current is  $\times 1/\sqrt{2}$ . For example, if  $I_1 = 1$  A and  $I_2 = 1$  A, the

current is  $I_c = 2$  A in the common mode expression and  $I_e = \sqrt{2}$  A in the even mode expression. Both the common mode and even mode represents the sum of signals, which means that they are qualitatively the same but quantitatively different. Different voltage and current makes the impedance (i.e. ratio) different. Since the voltage is  $\times\sqrt{2}$ , and the current is  $\times 1/\sqrt{2}$ , the impedances (i.e. ratio) differ by two times. (Note: The electrical power (obtained by multiplication) remains the same;  $\sqrt{2} \times 1/\sqrt{2} = 1$ .) Therefore,  $50 \Omega$  in the even mode equals to  $25 \Omega$  in the common mode. The relationship between the odd mode and the differential mode is the same:  $50 \Omega$  in the odd mode equals to  $100 \Omega$  in the differential mode. The reason why the impedance is  $25 \Omega$  (half) in common mode is that the common mode represents the parallel connection by definition. For example, the common mode characteristics for the two symmetrical transmission lines can be expressed as shown in Figure 2. Because  $50 \Omega$  lines are connected in parallel, the total impedance will be  $25 \Omega$ . Likewise, the definition of the differential mode represents the serial connection.

Figure 2 Two Symmetric Transmission Lines



These modes, which are slightly different from each other, have their own advantages and are used appropriately. Because the impedances do not change in the even and odd modes even after conversion, the use of these modes simplifies the theory construction. Therefore, even and odd modes are widely used in the microwave fields. On the other hand, because even and odd modes contain irrationals such as  $\sqrt{2}$ , they are a little more difficult to handle. Consequently, the common and differential modes are more popular in practical use. For example, if the characteristic impedance of a balanced cable is expressed as  $100 \Omega$ , it indicates the characteristic impedance in differential mode, not in odd mode.

### Common Mode is a Source of Radiative Noise

The current in differential mode is a reciprocal current in two conductors, and the backward route is positioned adjacent to the forward route (i.e. the loop area is small). Therefore, when viewed from a distance, forward and backward lines seem to compensate each other and no current flows. On the other hand, as mentioned above, the backward line of the current in the common mode is indeterminate and may be far from the forward line. Therefore, the loop area may be large and tend to be the source of radiative noise [13].

The common and differential modes are defined for systems such as two parallel conductor systems. Therefore, these modes can not be used for other situations such as the current distribution on a surface. However, in general, the term “common mode” may be used for expressing those situations in which due to the similarity the current deviates from the ideal reciprocal current (i.e. the loop area is large and can be the source of radiative noise). The “common mode” used in those situations is not defined clearly, and needs to be handled carefully to avoid misunderstandings.